$$(5)\times c-(3)\times 2ax,\ y=\frac{bcx-2abx-25c^2x}{bcx+4a^2x-2acx-c^2d}\dots\dots\dots\dots(6).$$

$$(5)\times b+(3)\times(25cx-bx),\ y=\frac{b^2x-50acx-bcd}{bcx-2abx-25c^2x}\dots\dots\dots\dots(7).$$

Now $b=25c-79a/4$ and $d=x^3+a$ in (6)=(7), etc.

$$(316a^2x-558acx)^2+(16a^2x-4c^2x^3-4ac^2-87acx+100c^2x)\times$$
$$(6241a^2x+316acx^3+316a^2c-400c^2x^3-400ac^2-16600acx+10000c^2x)=0\dots(8).$$

Expanding, we find a common factor c, then by substitution and reduction to the simplest form for the application of Horner's Method.

$$79x^{10}-1748x^9+12559x^8-2429.5x^7-478451.828125x^6$$
$$+2827762.5x^5-4080008.59375x^4-27582812.5x^3$$
$$+161863232.421875x^2-357007812.5x+301890625.=0\dots(9).$$

Horner's Method gives $x=7.95690209132$, (3) $y=0.564356664799$.
$x+y=8.521258756118$, $x-y=7.392545426520$.
$CE=3{,}521258756118$, $DE=4.392545426520$.
$\angle DAB=56°\ 17'\ 54''$, $\angle ABC=46°\ 11'\ 54''$, $\angle AEB=77°\ 30'\ 12''$.
$\angle DCE=59°\ 3'\ 32.5''$, $\angle CDE=43°\ 26'\ 15.5''$.

Tension on $BC=\dfrac{10\cos DAB}{\sin(DAB+ABC)}=5.6863$ pounds.

Tension on $AD=\dfrac{10\cos ABC}{\sin(DAB+ABC)}=7.0896$ pounds.

[NOTE. By a mistake we published the incomplete solution of this problem in our last issue. Soon after receiving that solution, Dr. Zerr wrote us to the effect that a correct and complete solution would shortly follow. Mr. Bell, being ill at the time, was unable to send the complete solution at the time expected, so that by the time the Department was ready for the press, we forgot about the promised complete solution and sent in the incomplete solution for publication. We have not verified the above solution, and our readers must excuse us from that great task. We hold Dr. Zerr and Mr. Bell responsible for any errors contained in it.—ED. F.]

71. Proposed by the late B. F. BURLESON, Oneida Castle, N. Y.

Three men own a sphere of gold the density of which varies as the square of the distance from the center. If two segments be cut off each one inch from the center of the sphere it will be divided into three parts of equal value. Determine the diameter of the sphere.

II. Solution by R. E. GAINES, A. M., Professor of Mathematics, Richmond College, Richmond, Va.

The element $dydx$ whose ordinate is y will, when revolved about the axis of x generate an infinitesimal ring whose volume is $2\pi ydydx$, and whose distance from the center is $\sqrt{(x^2+y^2)}$. Therefore for the mass of the minor segment we have

$$2\int_0^1\int_0^{\sqrt{(a^2-x^2)}}2\pi y(x^2+y^2)dydx=\pi\int_0^1(a^4-x^4)dx=\pi(a^4-\tfrac{1}{5}).$$

Mass of sphere $=\pi\int_0^a(a^4-x^4)dx=\frac{4}{5}\pi a^5$.

$\therefore\ \frac{4}{5}\pi a^5=3\pi(a^4-\frac{1}{5})$, $4a^5=15a^4-3$, which evidently has a root slightly less than 3.75.

In the solution given by Dr. Zerr in the November MONTHLY if the parts be added so as to give the mass of the sphere the result is not homogeneous in a and is therefore evidently wrong. In getting M the upper limit for θ should be $\cos^{-1}(1/r)$ and not $\cos^{-1}(1/a)$. For M_1 we must subtract $2M$ from the mass of the sphere.

75. Proposed by B. F. FINKEL, A.M.,M.Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

A particle P, is held in a bent tube by two forces directed towards two fixed points, H and S. Show that the equation of the tube is $PS.PH=k^2$, if the forces are μ/PS and μ/PH.

I. Solution by GEORGE R. DEANE, C. E., B. S., Professor of Mathematics, Missouri School of Mines, Rolla, Mo.

Put $PS=r_1$, $PH=r_2$. By the principle of virtual work, we have,

$$\frac{\mu}{r_1}\delta r_1+\frac{\mu}{r_2}\delta r_2=0.$$

Let $f(r_1,\ r_2)=0$ be the equation of the curve. Then

$$\frac{\partial f}{\partial r_1}\delta r_1+\frac{\partial f}{\partial r_2}\delta r_2=0.$$

Eliminating δr_1 and δr_2,

$$\frac{\dfrac{\partial f}{\partial r_1}}{\dfrac{\partial f}{\partial r_2}}=\frac{\dfrac{\mu}{r_1}}{\dfrac{\mu}{r_2}}.$$

Whence, $-\dfrac{dr_2}{dr_1}=\dfrac{r_2}{r_1}$, $r_1dr_2+r_2dr_1=0$, $r_1r_2=k^2$.

The general theorem of which this is a particular case, is given in Minchin's *Statics*, Vol. I., page 88.

II. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio State University, Athens, Ohio.

Let s=any arc of the tube, r, r'=the distances of the particle from the centers of force at any time t, m=the same absolute intensities of the forces, and β =the velocity of projection.

If S, S' be the radial forces attracting the particle, we will have